HYBRID MODEL ANALYSIS AND CONSTRUCTIVE ALGORITHMS IN THE PERFORMANCE OF A GENETIC ALGORITHM FOR THE PLANNING OF AN ELECTRIC POWER TRANSMISSION SYSTEM

ANTONIO ESCOBAR Z.  RAMON A. GALLEGO R.
Ing. Electricista     Ph.D. Ing. Eléctrica
Profesor Asociado   Profesor Titular
aescobar@utp.edu.co  ralfonso@utp.edu.co
UNIVERSIDAD TECNÓLOGICA DE PEREIRA-Pereira Colombia

RUBEN A. ROMERO L.  SÉRGIO A. DE OLIVEIRA
Ph.D. Ing. Eléctrica  M.Sc. Ing. Eléctrica
ruben@dee.feis.unesp.br  grilo@dee.feis.unesp.br
Professors at DEE-FEIS-UNESP-Ilha Solteira-Brasil

ABSTRACT

It has become more evident that in complex problems the performance of a genetic algorithm depends on the initial population, specially on its quality and diversity. In this paper four strategies are presented which, used concurrently allow to find quality and diversified topologies to set up an initial population to be used in a genetic algorithm to solve the problem of planning and expansion of electrical energy transmission systems. Also, experimental tests on real systems are presented to show the proposal's performance. This strategy to generate quality and diversified initial solutions can also be used in other algorithms such as simulated annealing, tabu search, GRASP, etc.

KEYWORDS: Planning, transmission system, hybrid model, genetic algorithm.

1. INTRODUCTION

Optimization algorithms such as simulated annealing, genetic algorithms, tabu search, GRASP, etc. usually require a solution or a set of solutions to start the search process. It can be shown in a theoretical analysis of these algorithms that the final solution does not generally depend on the initial solution used to start the process. This hypotheses is particularly useful in the theoretical form of the genetic algorithm. However, when an initial population is generated at random, the processing effort is prohibitive especially in real problems with a high degree of complexity.

Recent research on the application of genetic algorithms in complex problems show that an initial population generated by using efficient strategies considerably reduces the computational effort and improves the quality of the solutions found. It has also been verified that the best strategy to generate good initial solutions is to employ heuristic algorithms, usually of constructive type. One of the many reasons for this choice is that there are many heuristic algorithms available in the specialized literature to tackle complex problems. In this case, the proposal consists in finding a good quality and diversified population using constructive heuristic algorithms of relaxed models to solve the planning problem in electrical
transmission systems. The initial population found is used to start optimization using a genetic algorithm.

Relaxed models and heuristic techniques were widely used in the planning problem since the 60s and are still used. In this case two relaxed models called transport and hybrid are analyzed. With the hybrid model it is still possible to use three kinds of constructive heuristic algorithms. In this paper, a genetic algorithm with a new proposal to generate the initial population is used to carry out the planning of the Colombian system.

2. MATHEMATICAL FORMULATION

In the following, the static transmission expansion problem is formulated as a mixed integer nonlinear programming problem in which the power network is represented by a DC power flow model:

\[ \text{Min} \quad \nu = \sum c_{ij} n_{ij} \]

\[ \text{s.a.} \]

\[ S f + g = d \]

\[ f_{ij} - \gamma_{ij} (\theta_i - \theta_j) (n_{ij} + n^o_{ij}) = 0 \]

\[ | f_{ij} | \leq f_{ij \max} (n_{ij} + n^o_{ij}) \]

\[ 0 \leq g \leq g_{\max} \]

\[ 0 \leq n_{ij} \leq n_{ij \max} \]

where \( c_{ij} \) is the cost of the addition of a circuit in branch \( i-j \), \( n_{ij} \) is the number of the circuits added in branch \( i-j \), \( n^o_{ij} \) is the number of circuits in the initial system, \( S \) is the system's bus-branch incidence matrix, \( \gamma_{ij} \) is the susceptance of the new circuits, \( f_{ij} \) is the flow in the circuit \( i-j \), \( f_{ij \max} \) is the maximum flow in the circuit \( i-j \), \( d \) is the demand vector, \( g \) is the generation vector, \( g_{\max} \) is the maximum generation capacity vector.

A complication arises from the fact that the simple addition of a new circuit may not be enough to guarantee network connectivity. The resultant network can be disconnected.

The DC model must obey Kirchhoff's two laws and the operation and investment constraints. Thus, the first set of restrictions in (1) represents Kirchhoff's first law and the second set, Kirchhoff's second law. The rest are operational restrictions. Problem (1) is a non-linear programming integer mixed problem (PNLIM). It is hard to solve due to its non-convexity and to the fact their shows the combinatorial explosion phenomenon. One of the most appropriate ways to solve (1) is by means of a genetic algorithm.

3. RELAXED MODELS AND HEURISTIC ALGORITHMS

The DC model can be substituted by relaxed models where a set of restrictions are not taken into account, making it possible to solve the problem more easily. The most widely used
relaxed mathematical models are the transport models and the hybrid models. The latter can have several forms and it can be linear or non-linear.

3.1 TRANSPORT MODELS

When the restrictions corresponding to Kirchoff's second law in the DC model are eliminated, a relaxed model called "Transport model" is obtained. It is governed by the following equations:

\[
\begin{align*}
\text{Min} & \quad \mathbf{v} = \sum c_{ij} n_{ij} \\
\text{s.a.} & \quad S f + g = d \\
& \quad |f_{ij}| \leq f_{ij \max} (n_{ij} + n^o_{ij}) \\
& \quad 0 \leq g \leq g_{\max} \\
& \quad 0 \leq n_{ij} \leq n_{ij \max}
\end{align*}
\]

\(n_{ij}\) integer; \(f_{ij}\) unbounded

The transport model corresponds to a linear integer mixed problem (PLIM). It is not easy to find the optimal solution to the transport model. However, there is a constructive heuristic algorithm that finds a good solution to the transport model with a small computing effort. A good solution found with the heuristic algorithm is an excellent topology to build an initial population because it generally finds the most attractive circuits to be incorporated into the system.

3.2 LINEAR HYBRID MODEL

This is also a relaxation of the DC model. In the linear hybrid model all existing circuits in the base topology must fulfill both of Kirchhoff's laws and the added circuits must only comply Kirchhoff's first law. In other words, Kirchhoff's first law must be obeyed in all the system's buses but the second law must only be obeyed in the meshes of the already existing circuits in the base configuration. A mathematical formulation for the linear hybrid model is:

\[
\begin{align*}
\text{min} & \quad \mathbf{v} = \sum c_{ij} n_{ij} \\
\text{s.a.} & \quad S f + S^o f^o + g = d \\
& \quad f^o_{ij} \cdot \gamma_{ij} n^o_{ij} (\theta_i - \theta_j) = 0 \\
& \quad |f^o_{ij}| \leq f_{ij \max} n^o_{ij} \\
& \quad |f_{ij}| \leq f_{ij \max} n_{ij} \\
& \quad 0 \leq g \leq g_{\max} \\
& \quad 0 \leq n_{ij} \leq n_{ij \max}
\end{align*}
\]

\(n_{ij}\) integer; \(f^o_{ij}\), \(f_{ij}\) y \(\theta_j\) unbounded

Where \(S^o\) is the system's bus-branch incidence matrix conformed by the existing buses and branches in the initial topology, \(f^o\) is the power flow vector of the existing branches in the
initial topology, \( f \) is the flow vector of the newly added circuits whose elements are the flows \( f_{ij} \).

### 3.3 HEURISTIC ALGORITHMS

Heuristic constructive algorithms exist for each of the presented models. Those algorithms add a circuit in every step and the circuit is selected by using a sensitivity indicator. It is important to note that if in the transport and linear hybrid models the integrality conditions are relaxed, that is, the values \( n_{ij} \) are allowed to take non-integer values, then both models reduce to solving a simple linear programming problem (PL). This kind of solution has no practical value. However, it can be used as a strategy to identify the most attractive circuits, that is, it can be used as a sensitivity indicator. Thus, for the transport and hybrid models the following heuristic constructive method can be used:

1. Consider the initial topology as the present topology.

2. Solve the mathematical model for the present topology using a PL algorithm, relaxing the integrality of values \( n_{ij} \). If all values \( n_{ij} = 0 \) and \( \nu = 0 \) then stop the process since the system operates adequately. Else, go to step 3.

3. Identify the most attractive path as the added circuit that takes the greatest power flow \((n_{ij}>0)\). Add this circuit to the simulated system and update the present topology.

This algorithm finds a feasible topology for the analyzed model. In large sized systems heuristic algorithms do not find the optimal solution. Also, the found topologies, although optimal, aren't usually feasible for the DC model. However, the algorithm finds good solutions and, even more important, it identifies the most attractive circuits. Therefore, the fundamental strategy consists of finding a set of topologies using relaxed models and then employing those topologies as an initial population to find optimal and sub-optimal solutions for the DC model combined with a specialized genetic algorithm.

### 3.4 FOUR HEURISTIC ALGORITHMS

Using the already discussed proposal of a constructive heuristic algorithm it is possible to implement an algorithm for the transport model and three algorithms for the hybrid model. With small modifications in the basic algorithm it is possible to find a set of good quality topologies. The figure 1 shows the solution trajectories for the hybrid and transport models.

#### 3.4.1 GARVER'S ALGORITHM

The heuristic algorithm developed in 3.3 for the transport model is Garver's proposal [6] and it was widely used in the past for planning. Garver's algorithm finds a feasible topology for the transport model. However, many topologies can be found using Garver's algorithm. For example, suppose that Garver's algorithm finds a topology where circuits where added in 40
paths and 25 from those paths are considered important. In this context, 25 further topologies can be found repeating Garver's algorithm and discarding in each case one of the 25 paths considered important. This implementation can be made easily by changing the cost of each circuit in every case.

The foregoing process can be repeated discarding several circuits in each case. It is important to note that many good quality topologies can be found and they can be significantly different.

![Figure 1. Solution trajectories of the hybrid and transport models](image)

**3.4.2 HYBRID ALGORITHM I**

It is possible to employ the heuristic algorithm of 3.3 for the hybrid model where the final topology is feasible for the hybrid model. In this case, in step 2 the hybrid model (3) is solved applying the following changes:

1. Relation \( S \mathbf{f} + S^o \mathbf{f}^o + \mathbf{g} = \mathbf{d} \) (Kirchhoff's first law is substituted by relation \( S \mathbf{f} + S^o \mathbf{f}^o + S' \mathbf{f}' + \mathbf{g} = \mathbf{d} \), where \( S' \) is the bus-branch incidence matrix formed by the circuits added during the heuristic process and the terminal buses in those circuits. \( \mathbf{f}' \) is the flow vector with elements \( f'_{ij} \) which represent the circuit flow through path \( i - j \) that was added during the iterative process.

2. The set of relations \( |f'_{ij}| \leq f_{ij\text{max}} n'_{ij} \) must be added, where \( n_{ij} \) represents the number of circuits added during the iterative process.

It is important to note that every time that the modified problem (3) is solved, there is a flow vector for circuits in the base topology and another flow vector for circuits added in the iterative process and yet another flow vector for circuits with values \( n_{ij} \neq 0 \) in the PL solution. It is also important to note that the modified problem (3) changes little between 2 successive iterations since vector \( n' \) scarcely varies with the addition of a circuit in element \( n'_{ij} \) on the most attractive path \( i-j \). Matrix \( S' \) also changes.
3.4.3 HYBRID ALGORITHM II

It is possible to use the heuristic algorithm shown in 3.3 for the hybrid model where the final topology has characteristics that are slightly different from those found in the last proposal. In this proposal, every circuit added during the iterative process must show the following behavior: (1) if it belongs to a path where a circuit already exists in the base configuration, then the newly added circuit must obey Kirchhoff's second law together with the parallel circuits that may have existed on that path in the base topology and (2) if it belongs to a new path then that circuit must only obey Kirchhoff's first law. Thus, the topology found under this proposal is different from the one found in the last proposal. Suppose for example that a circuit exists between buses 25-32 in the base topology and during the iterative process a new circuit on that path was added. In the hybrid algorithm I it is possible that the flow through each of the two circuits be different (one of them fulfills only Kirchhoff's first law, the other fulfills both laws). In the hybrid algorithm II both circuits must have equal flows since the two must fulfill both Kirchhoff's laws.

In this case, in step 2 the hybrid model (3) is solved with the following changes (is used the model of hybrid I):

On each iteration it is identified if the circuit to be added belongs to a path on which at least a circuit of the base topology exists. That being the case, element $n_{ij}^o$ is updated adding a circuit. Else, element $n_{ij}'$ is updated adding a circuit. An update of matrix $S'$ may also be necessary. It is important to note that the PL structure between two successive iterations changes little; only an element from $n_{ij}^o$ or an element from $n_{ij}'$ (and also $S'$).

3.4.4 HYBRID ALGORITHM III

It is possible to use the heuristic algorithm of 3.3 for the hybrid model where the final topology is feasible for the DC model. This proposal is known as the Villasana-Garver algorithm. In this case, step 2 of the hybrid model (3) is solved as follows:

1. During each step, relaxed problem (3) is solved using a PL algorithm.

2. On each iteration, the most attractive path $i$-$j$ must be identified and element $n_{ij}^o$ must be updated adding a circuit. It may also be necessary to update matrix $S^o$.

It is important to note that on each step of the iterative process, the circuits existing in the base topology and also those added in the iterative process obey Kirchhoff's second law. Therefore, when the iterative process ends, the final topology found is feasible for the DC model.

3.5 GENERATION OF THE INITIAL POPULATION

The initial population, conformed by excellent-quality and diversified topologies, can be found using the four constructive heuristic algorithms shown above. It is also observed that constructive heuristic algorithms tend to add many irrelevant circuits in the final phase of the
iterative process. Therefore, some topologies can be generated by stopping the process when
the investment $\nu$ found by the PL algorithm is small enough without reaching $\nu=0$. In this
work, topologies to find an initial population were generated according to the following steps:

- For each heuristic algorithm an additional set of new topologies can be found by repeating
  the process and forbidding in each case an important circuit identified in the initial process
  with no prohibitions.
- The aforementioned strategy can be repeated, but the process stops when the pending
  investment $\nu$ is $\nu \leq \nu_0$, where $\nu_0$ is a value previously fixed.

With this proposal it is possible to find a high number of topologies, usually much higher than
the necessary to set up the initial population of a genetic algorithm. However, new good-
quality topologies can be found by employing the recombination operator on the found
topologies.

4. GENETIC ALGORITHMS

The methodology described for a genetic algorithm is taken from reference [3]. Several
methologies of initializing algorithms are tested using the same base algorithm.
A simplified representation of a Genetic Algorithm Problem requires the following steps:

- Definition of the objective function
- Coding
- Definition of the selection scheme
- Definition of the crossover and mutation mechanisms
- Definition of population size and stopping criterion
- Implementation of the special features associated to the transmission planning problem

In order to develop a genetic algorithm capable of solving large-scale practical cases it is
necessary to implement the following functions: Limited selection, elitism, building blocks
and unconnected networks, see reference [3].

Control parameters: The parameters used in the analysis of the colombian electrical system
are: population size between 140-200, crossover rate between 0.7-0.9, mutation rate between
0.005-0.05. The selection type used is proportional selection with stochastic remainder.

Stop criterion: The process stops whenever the incumbent solution (least cost configuration)
does not improve after a specified number of generations (from 50 to 100 generations). The
maximum number of generations used is between 500 and 1000.

5. TEST AND RESULTS

Using the Minos program (running in a SUN ultra5 station) a genetic algorithm was
implemented, with an initial population generated with the three versions of the hybrid model
and the transport model. As test systems the Garver system (6 buses/15 branches) [6] and the
Colombian electrical system projected into year 2012 (171 buses/ 223 existing transmission
lines and 196 candidates) were used. All the relevant data about this system are found in reference [1].

In the Garver system [6] hybrid initializers complemented with the constructive algorithm described in 3.3 find the optimal solution without resorting to a genetic algorithm. It was found however that each hybrid version reaches the solution through different solution trajectories as shown in figure 2.

The constructive heuristic techniques used in this paper allow the obtention of an optimal solution for small systems.

For the 2012 colombian system, the obtained solutions were confronted using the constructive heuristics described in 3.3 based on the three hybrid versions with the best solution found to date for this system [5]. Figure 3 shows the circuits common and not common to the solutions found with hybrid versions and the best solution in reference [5].

Based on hybrid and transport models an initial population was set up and then used with a genetic algorithm. The following solution was obtained:


With a total investment of US M562.3 and a loss of load of 0 MW, the same quality as that reported in [1,5] was reached. Also, other good solutions were found, one of this with a total investment of US M504.37 and a loss of load of 50 Mw.

6. CONCLUSIONS

In this paper, four constructive algorithms are analyzed: the transport model and 3 versions of the hybrid model. This paper’s objective was to conform initial populations with the forementioned algorithms and to use those populations as initializers for combinatorial algorithms such as genetic, memetic, evolutive, tabu search, simulated annealing and GRASP algorithms. The population obtained was used as initializer for a genetic algorithm.
The solution obtained was substantially better as the one achieved with the employment of other initializers using the same genetic algorithm and was as good as that obtained using a tabu search.

It can be shown mathematically that it is possible to find very high quality solutions in large electrical systems, but the computational effort is prohibitive in terms of time. In practice, with a random or some other kind of initializer, usually very poor quality solutions are obtained and still the computational effort is high. When the methods described in this paper to find an initial population are used, it is guaranteed that the process starts in a region that contains very good quality solutions, among which the optimal solution may be included. The computational effort is moderate with respect to that made under some other initializer.

7. ACKNOWLEDGEMENTS

The authors wish to thank to the Universidad Tecnológica de Pereira for the support to the electric planning work group.

8. BIBLIOGRAPHY


