Multi-stage and Coordinated Planning of the Expansion of Transmission Systems

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Abstract—In this paper an efficient genetic algorithm is presented to solve the problem of multi-stage and coordinated transmission expansion planning. This is a Mixed Integer Non-Linear Programming (MINLP) problem, difficult for systems of medium and large size and high complexity. The Genetic Algorithm (GA) presented has a set of specialized genetic operators and an efficient form of generation of the initial population that finds high quality sub-optimal topologies for large size and high complexity systems. In these systems, multi-stage and coordinated planning present a lower investment than static planning. Tests results are shown in one medium complexity system and one large size high complexity system.

Keywords: Multi-stage planning, optimization, network expansion planning, genetic algorithms.

I. INTRODUCTION

The mathematical model for transmission system expansion planning problems is NP-complete, that is, a problem for which no method exists that solves it in polynomial time. This problem presents a large number of local optimal solutions and when system size becomes large, the number of solutions grow exponentially. A solution to a planning problem specifies where, how many, and when new equipment must be installed in an electric system, so that it operates adequately within a specified planning horizon.

The objective of expansion planning is to determine the bulk power system that is able to meet the forecast demand at the lowest cost while satisfying prescribed technical, financial and reliability criteria. This process is typically broken down into the following two stages: Long term transmission expansion planning (LTTEP) and mid-term transmission expansion planning (MTTEP).

The models that this paper analyzes apply to the LTTEP problem. There are a number of considerations (e.g., those related to transient stability limits, voltage violations, reactive power flows, short-circuit capacity, etc.) that cannot be easily taken into account in the LTTEP.

Traditionally, transmission planning is solved in two ways. The simpler one is static planning; it considers only one planning horizon and determines the number of circuits that should be added to each branch of the electric system. Investment is carried out at the beginning of the planning horizon time. The second one is multi-stage and coordinated planning, that defines not only optimal locations and type of investments, but also the most appropriate times to carry out such investments, so that the continuing growth of the demand and generation is always assimilated by the system in an optimized way. The planning horizon is divided into several stages and the circuits must be added to each stage of the planning horizon. Investment is carried out at the beginning of each stage. When multiple stages in the optimization process are considered, the objective is the minimization of the present value of the sum of all the investments carried out throughout the years corresponding to the simulated periods.

Currently, the available bibliography on transmission systems expansion planning analyzes only the problem of static planning, which is already difficult to solve for complex systems. In multi-stage and coordinated planning, the planning horizon is separated into several stages and proposals for the addition of circuits that optimize the investment in the complete planning horizon, must be determined for each period. This problem is obviously harder. In this paper we analyze the multi-stage and coordinated planning problem.

Several mathematical models are used in the expansion planning problem, but the best accepted is the so-called DC model. In many articles, relaxed versions of the DC model (transportation and hybrid models) are used and the AC model is generally used in the following planning stages. In this article we use the DC model in the genetic algorithm and the relaxed models as part of the specialization strategy of the GA to generate a high quality initial population. The solution techniques used in the past were diverse but may be classified in 3 groups: (1) constructive heuristic methods, (2) classic optimization and (3) intelligent systems. The latter has been used in recent years because it has the capacity to find good or sub-optimal solutions in large complex systems. Algorithms such as simulated annealing (SA) [15], genetic algorithms (GA)[11], [12], tabu search (TS)[13], greedy randomized adaptive search procedure (GRASP) [14] and hybrid versions belong to this group of methods. In this article a specialized GA is developed for the multi-stage and coordinated planning problem.

The proposed model is more detailed than the traditional one used for the transmission system expansion planning problem (TSEP). The reason is that it allows an integrated multi-stage planning of generation and transmission systems dealing with operation costs. However, in this work only the transmission planning is addressed. The results are compared with the ones obtained in the static planning model. The detailed analysis of an integrated planning of generation and transmission systems is still in development and represents a promising research area.

It is worth observing that the transmission system expansion planning is too complex to be fully analyzed in a single pa-
The objective function considers the investment in generation where the model proposed in [10] is outlined, in which operation is separated for each year of this horizon. Considering an annual discount rate of \( r \), the results obtained in large size and high complexity test systems are presented, and, finally, a number of conclusions are presented.

This paper is organized as follows: initially the mathematical formulation of the static and multi-stage and coordinated planning problems is presented. Then, the genetic algorithm for the multi-stage and coordinated planning problem is analyzed; next, the results obtained in large size and high complexity test systems are presented, and, finally, a number of conclusions are presented.

II. MATHEMATICAL FORMULATION OF THE STATIC AND MULTI-STAGE AND COORDINATED PLANNING PROBLEM

In this section a mathematical model of a more general character than the model proposed in [10] is outlined, in which the objective function considers the investment in generation and transmission as well as the system operational costs for the planning horizon.

The static model considers that the investment is carried out in the initial year of the planning horizon and that the operation is separated for each year of this horizon. Considering an annual discount rate of \( I \), the present values of the investment costs and operation for the base year \( t_0 \), with an initial year \( t_1 \) and with a horizon of \( t_2 - t_1 \) years are the following:

\[
c(x) = (1 - I)^{t_1 - t_0} c_1(x) = \delta_{inv} c_1(x) \quad (1)
\]

\[
d(y) = (1 - I)^{t_1 - t_0} d_1(y) + (1 - I)^{t_2 - t_1} d_1(y) + \ldots + (1 - I)^{t_2 - t_1} d_1(y) = \delta_{oper} d_1(y) \quad (2)
\]

where

\[
\delta_{inv} = (1 - I)^{t_1 - t_0}, \quad \delta_{oper} = \sum_{t=t_1}^{t_2-1} (1 - I)^{t-t_0} \quad (3)
\]

\( c_1(x) \) is the investment and \( d_1(y) \) is the yearly operational cost.

The DC model, including generation and transmission planning as well as the investment and operational costs, can be written in the following form [10]:

\[
\min \nu = \delta_{inv} \left( \sum_{(i,j)} c_{ij} n_{ij} + \sum_i C_i N_i \right) + \delta_{oper} \left( \sum_i O C_i G_i + \sum_j \alpha \sum_k r_k \right) + \delta_{oper} \left( \sum_i O C_i G_i + \sum_j \alpha \sum_k r_k \right)
\]

s.t.

\[
\begin{align*}
B \theta + G + g + r &= d \\
(n_{ij} + n_{ij}^o) [\theta_i - \theta_j] &\leq (n_{ij} + n_{ij}^o) \varphi_{ij} \\
N_i G_i &\leq G_i \leq N_i G_i \\
g_j &\leq g_j \leq \bar{g}_j \\
0 &\leq r \leq d \\
\underline{p}_{ij} &\leq n_{ij} \leq \overline{p}_{ij} \\
N_i &\leq N_i \leq \overline{N}_i \\
n_{ij} &\in N_i \text{ integer} \\
\theta_i &\text{ unbounded}
\end{align*}
\]

where \( v \) is the present value of the expansion and operation cost (generation and transmission) of the system, \( \delta_{inv} \) is the discount factor to find the present investment value, \( c_{ij} \) is the cost of a circuit in the \( i - j \) branch where \( n_{ij} \) circuits were added, \( n_{ij}^o \) are the branches in the initial configuration, \( C_i \) is the installation cost of a candidate generator \( i \) where \( N_i \) generators were added, \( \delta_{oper} \) is the discount factor modified to take into account the duration (in years) of the planning horizon, \( OC_i \) is the annual operation cost of the candidate generator \( i \) that injects an active power \( G_i \), \( oc_j \) is the annual operation cost of the generator \( j \) already installed and that injects an active power \( g_j \), \( \alpha \) is the factor to commoditize cost units with loss of load, \( r \) is the loss of load vector in buses with elements \( r_k \), \( B \) is the admittance matrix of the initial network and candidate circuits, \( G \) is the vector of active power injections of the candidate generators, \( g \) is the vector of active power injections of the generators already installed, \( \theta \) is the vector of angles of the buses with components \( \theta_i, \varphi_{ij} \) is the maximum angle between the \( i \) and \( j \) nodes and \( d \) is the demand vector. \( \overline{N}_i, \underline{p}_{ij}, \overline{g}_j \) and \( \underline{G}_i \) represent these variables upper limits and \( N_i, \overline{N}_i, \underline{p}_{ij}, \overline{g}_j \) and \( \underline{G}_i \) represent the lower limits.

In system (4) the investment variables are represented by the number of generators \( N_i \) and by the number of circuits (transmission lines and transformers) \( n_{ij} \) that must be installed. The operation variables are represented by the existent generators injection of active power, \( g_j \), and candidates, \( G_i \). The variables \( r_k \) are artificial generation variables that may be zero and can be interpreted as costs by loss of load.

In multi-stage and coordinated planning, the planning horizon is divided into several stages, for example in five year-long stages, and in that context the equipment that should be installed in every planning stage should be determined. Considering an annual discount rate \( I \), the present values of the investment costs and operation costs, for the reference year \( t_0 \), with an initial year \( t_1 \), with a horizon of \( t_2 - t_1 \) years, and with \((T-1)\) stages, are the following:

\[
\begin{align*}
\min \nu &= (1 - I)^{t_1 - t_0} c_1(x) + (1 - I)^{t_2 - t_0} c_2(x) + \ldots + (1 - I)^{t_T - t_0} c_T(x) \\
&= \delta_{inv}^1 c_1(x) + \delta_{inv}^2 c_2(x) + \ldots + \delta_{inv}^T c_T(x)
\end{align*}
\]
The problem (5) is a MINLP with a significant increment of the variables, in relation to the static planning problem. The analysis of an efficient algorithm to solve the complete model of the planning problem (5) will be presented in another paper. This article analyzes only a particular case of problem presented in (5), whose aspects related to the generation expansion and operation costs are not taken into account. In this condition, the problem (5) becomes the following multi-stage transmission planning problem:

\[
\min v = \sum_{t=1}^{T} \left[ \delta_{\text{inv}}^t \left( \sum_{(i,j)} c_{ij} n_{ij}^t + \sum_{i} C_i N_i^t \right) + \delta_{\text{oper}}^t \left( \sum_{i} O C_i G_i^t + \sum_{j} o c_j g_j + \alpha \sum_{k} r_k^t \right) \right]
\]

s.t.
\[
B^t \theta^t + G^t + g^t + r^t = d^t
\]
\[
(n_{ij}^m + \sum_{m=1}^{t} n_{ij}^m) (|\theta_i^t - \theta_j^t|) \leq (n_{ij}^o + \sum_{m=1}^{t} n_{ij}^o) \phi_{ij}^t
\]
\[
\sum_{m=1}^{t} N_i^m G_i \leq G_i \leq \sum_{m=1}^{t} N_i^m C_i
\]
\[
\theta_i^t \leq \theta_j^t \leq \theta_i^t
\]
\[
0 \leq r^t \leq d^t
\]
\[
0 \leq n_{ij}^t \leq m_{ij}^t
\]
\[
N_i^t \leq N_i \leq \bar{N}_i
\]
\[
\sum_{t=1}^{T} n_{ij}^t \leq m_{ij}
\]
\[
\sum_{t=1}^{T} N_i^t \leq \bar{N}_i
\]
\[
n_{ij}^t e N_i^t \text{ integer}
\]
\[
\theta_i^t \text{ unbounded}
\]
\[
t = 1, 2, \ldots, T.
\]

For problems that use model (6), only metaheuristics like GA, SA, TS, GRASP, etc., have the capacity to find high quality solutions of real systems. In this article a specialized GA is presented.

III. EFFICIENT GENETIC ALGORITHM IN EXPANSION PLANNING

In this section the basic concepts of the GA are presented, as well as the structure of a specialized GA algorithm for the multi-stage and coordinated transmission systems planning problem.

A. Basic Foundations of Genetic Algorithms

For complex problems like transmission expansion planning, the more successful algorithms in relation to the quality of the solution reached are combinatorial algorithms or metaheuristics that have the capacity to find high quality solutions to complex problems. These algorithms are SA, GA, TS, GRASP, etc.

The fundamental strategy of a combinatorial algorithm consists in carrying out a set of transitions, in an efficient way, by means of interesting solutions to the problem. These transitions guide the process to optimal or high quality sub-optimal solutions. The GA analyzes in an intelligent way, a very reduced number of proposed solutions with a strategy that allows finding high quality solutions. This contrasts with constructive heuristic algorithms where a set of transitions is also carried out, but a unique solution is found that is a local optimum, and

\[
d(y) = \sum_{t=1}^{t_2} (1 - I)^{t-t_n} d_1(y) + \sum_{t=t_2}^{t_3} (1 - I)^{t-t_n} d_2(y)
\]
\[
+ \ldots + \sum_{t=t_p}^{t_{p+1}} (1 - I)^{t-t_n} d_T(y)
\]
\[
= \delta_{\text{inv}}^t d_1(y) + \delta_{\text{oper}}^t d_2(y) + \ldots + \delta_{\text{oper}}^T d_T(y)
\]

where \( \delta_{\text{inv}}^t = (1 - I)^{t-t_n} \) and \( \delta_{\text{oper}}^t = \sum_{p=t_n-t_n}^{t_{p+1}-1} (1 - I)^p \)

and \( d_1(y) \) is the yearly operation cost of each stage.

With the previous equation, the DC model for the multi-stage and coordinated planning problem, considering generation and transmission expansion as well as operation costs, assumes the following form [10]:

\[
\min v = \sum_{t=1}^{T} \left[ \delta_{\text{inv}}^t \left( \sum_{(i,j)} c_{ij} n_{ij}^t + \sum_{i} C_i N_i^t \right) + \delta_{\text{oper}}^t \left( \sum_{i} O C_i G_i^t + \sum_{j} o c_j g_j + \alpha \sum_{k} r_k^t \right) \right]
\]

s.t.
\[
B^t \theta^t + G^t + g^t + r^t = d^t
\]
\[
(n_{ij}^o + \sum_{m=1}^{t} n_{ij}^m) (|\theta_i^t - \theta_j^t|) \leq (n_{ij}^o + \sum_{m=1}^{t} n_{ij}^o) \phi_{ij}^t
\]
\[
\sum_{m=1}^{t} N_i^m G_i \leq G_i \leq \sum_{m=1}^{t} N_i^m C_i
\]
\[
\theta_i^t \leq \theta_j^t \leq \theta_i^t
\]
\[
0 \leq r^t \leq d^t
\]
\[
0 \leq n_{ij}^t \leq m_{ij}^t
\]
\[
N_i^t \leq N_i \leq \bar{N}_i
\]
\[
\sum_{t=1}^{T} n_{ij}^t \leq m_{ij}
\]
\[
\sum_{t=1}^{T} N_i^t \leq \bar{N}_i
\]
\[
n_{ij}^t e N_i^t \text{ integer}
\]
\[
\theta_i^t \text{ unbounded}
\]
\[
t = 1, 2, \ldots, T.
\]
in the case of large and complex problems, generally of poor quality. Also, this strategy contrasts with the classic techniques of optimization where the optimal or sub-optimal solution is generally obtained only after the algorithm converges. An additional advantage of genetic algorithms is that they are robust, that is to say, they always converge and generally lead to sub-optimal solutions of complex problems. The main disadvantage is that GAs generally need relatively long processing times but this is not a problem in transmission expansion planning.

The fundamental difference between various metaheuristics is the way transitions are carried out, that is, in a given current optimization where the optimal or sub-optimal solution is found. This set of solutions is subject to a process of selection, recombination and mutation, and then a new population is generated. This new population is designated as the current population. The GA begins the process of optimization with an initial population and applies the genetic operators to find new generations. In GA the following aspects should be specified: (1) the way in which the information is coded, which is fundamental in the determination of the algorithm quality, (2) the genetic parameters such as the population’s size and recombination and mutation rates, (3) the implementation of the genetic operators of selection, recombination and mutation, (4) the way in which the initial population is generated and (5) the structure of new genetic operators and/or strategies that consider the particular characteristics of the problem. The basic theory in genetic and evolutionary algorithms is described in [11], [12].

In the implementation process of the genetic operators, initially the form of codification of the solutions should be selected. This decision is critical in the formulation of a genetic algorithm for a specific problem, because it determines the functioning of the genetic operators. Planning is a problem with continuous and integer variables, and practically in all the proposals of genetic algorithms projected for the planning problem, the integer variables are coded and the continuous variables are found, in an exact way, solving a Linear Programming (LP) problem [6], [16]. With this strategy, it is not necessary to code the real variables but it does demand the use of a LP algorithm that consumes the largest percentage of the execution time of the genetic algorithm. GAs applied to the static planning problem can be found in [6], [8], [9], [16].

**B. Genetic Algorithm in Expansion Planning**

The specialized literature reports genetic algorithms with several selection methods, several mutation proposals and recombination of several points; some of these issues have been developed by the authors in [7], [8]. Another strategy used in the GA is presented in [6] and consists in generating the GA initial population using efficient constructive heuristic algorithms. In this way, it is possible to find diverse solutions of high quality to begin the genetic algorithm and for the feedback of the process, storing the best configurations as elite solutions. A GA for the planning problem should specify the following: (1) the chosen codification, (2) the implementation form of the genetic operators, (3) the choice of certain genetic parameters, (4) the way of generating the initial population, (5) the new applied genetic operators using specialized strategies, that take advantage of the specified characteristics of the problem and (6) other additional aspects of implementation. Next, the aspects mentioned previously for the GA used in the multi-stage and coordinated planning problem are presented.

The proposed codification for the application of a GA to a specific problem, is the most crucial aspect of the structure of the GA. The codification facilitates or complicates the implementation of the GA mechanisms. In this paper a decimal code for the investment variables is used. In this way, for example, one configuration of a system that has $n_t$ paths to add circuits and $t$ planning stages, should be represented by a vector of size $n_t \times t$. This vector should be divided into $t$ sectors and each sector must contain the number of circuits added in each planning stage. Figure 1 shows a topology represented by vector $P_a$ from a system with $n_t = 183$ paths and two planning stages $P_{a1}$ and $P_{a2}$. In this example, if 2 circuits in path $1 - 2$ are added to the first stage and one circuit added to the second stage, the circuits added to the first stage are automatically used in the second stage. This way, in the $P_{a2}$ period, 3 circuits operate in path $1 - 2$.

Once the codification form for one topology is defined, the objective function of this topology should be found. For the proposed codification, the objective function is obtained by solving a LP problem, which solves the $t$ planning stages in a single plan in a simultaneous and coordinated way. It should be observed that once an investment proposal is defined, with values known for $n_{t,j}$ in each planning stage, then the problem (6) is simply a LP problem with continuous variables. Obviously, in the solution to the LP, for stage $t$, the circuits of the base topology should be considered as well as the circuits added in previous stages; that is, in stages $1, 2, \ldots, (t - 1)$. The calculation of the objective functions of the topologies demands most of the processing time of the GA in the multi-stage and coordinated problem, because it is necessary to solve LP problems.

<table>
<thead>
<tr>
<th>recombination point</th>
<th>$P_{a1}$</th>
<th>$P_{a2}$</th>
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</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>1-2</td>
<td>1-0</td>
</tr>
<tr>
<td>2-4</td>
<td>0-1</td>
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<tr>
<td>2-60</td>
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<td>$\ldots$</td>
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<td>80-83</td>
<td>81-83</td>
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<td>82-84</td>
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<td></td>
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<tr>
<td>$T_2$</td>
<td>2</td>
<td>2</td>
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<td>2</td>
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<td>183</td>
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</tbody>
</table>

**Figure 1:** Codification in multi-stage and coordinated planning
tion process, all topologies are considered as feasible and the optimization multi-stage eliminates those topologies with loss of load.

Once the population is defined, the GA applies the selection, recombination and mutation operators. There are several proposals on how to implement selection but most of the selection methods work in a similar way in a planning problem. Therefore, the fast and efficient method called tournament selection is used. This proposal carries out $n_p$ games for a population size $n_p$. In each game, the algorithm chooses $k$ topologies randomly and the winning topology (with the right to generate a descendant) is that whose objective function is the best. This is an efficient proposal when a small value for $k (k = 2, 3, 4)$ is chosen and a maximum number of descendants for each topology is defined. Additionally, the tournament selection does not have the problems that appear in both proportional selection and methods derived from it.

The recombination operator can apply recombination at several points. The most important aspect in the recombination applied to the multi-stage and coordinated planning problem consists in reviewing the information that exists in a path, for each planning stage, for each descendant. For example, in Figure 1, for the recombination point indicated, the circuits added in paths 1 – 2 and 2 – 4 in the two planning stages should be reviewed for each descendant. This particularity indicates that it is more appropriate to separate the code of one topology into $t$ vectors as shown in Figure 1.

The mutation operator is easily implemented when decimal codification is used. For this reason, this codification method was used in the problem of multi-stage and coordinated planning in a similar way to that of static planning. This way, once a mutation point, or path for addition or subtraction of new circuits is chosen, the decision to increase or decrease the number of circuits in one unit should be taken randomly. Obviously, if the number of circuits in the selected branch is zero, then the decision consists in adding circuits, and if the number of circuits in a branch is the maximum permitted, then the decision consists in eliminating circuits. Another strategy consists in adding two or more circuits. This represents a larger mutation.

The parameters of the GA used are presented along with the results of the tests of the systems analyzed. The way of generating the initial population as well as the new genetic operators and specialized implementations in the GA, are analyzed in the following subsections.

C. Efficient Generation of the Initial Population

GA presents a better performance, in relation to processing time and solution with excellent objective function value, when a good initial population is used [6], [8]. This performance is radically different when large and complex systems are analyzed.

An interesting strategy to generate a good initial population consists in using configurations obtained with constructive heuristic algorithms (CHA) [1], [2], [3], [4]. A topology found by CHA can be considered good if it presents one or both of the following characteristics: (1) it has an excellent objective function value, that is, it represents a good investment proposal and (2) the topology has a large number of circuits that form part of the optimal topology. The two characteristics do not always appear simultaneously in the same topology in the planning problem.

The initial population of a GA can be considered as excellent if the circuits that form part of the optimum topology are present in the different topologies of the initial population. In this case, the selection and recombination operators can combine those optimum circuits together into a unique topology, transforming it into an optimum topology. Therefore, if the number of circuits in each path of the optimum topology is not present in the population, the only way to build the optimum topology is by means of the mutation operator, which is considered a secondary operator in the GA. For large-sized systems in the planning problem, the randomly-generated initial populations have a small number of optimum and sub-optimum circuits within the initial population. Accordingly, mutation must generate those optimum circuits, making the GA computationally unviable. In this paper, we use the relaxed models and a CHA for generating an initial population for the GA such that the initial population has a large number of components from the optimum and sub-optimum solution. For more details on the use of relaxed models and CHA for generating an initial population, see reference [8].

Relaxed models and CHAs can be used for generating quality topologies, particularly those in which the topology generated has many circuits that make up part of the optimum or sub-optimum topology.

In this paper, the CHAs presented in [1], [2], [3], [4] were implemented. These algorithms are used in different network models (transportation, hybrid, and DC).

For each mathematical model and for each CHA, the following algorithm can be used to generate a set of different topologies:

1) Generate the best topology for the mathematical model selected and for the CHA chosen. Suppose that circuits were added in $p$ paths and that we wish to find $k \leq p$ different topologies. Sort the $p$ circuits in descending order of cost.

2) Using the information from each of the first $k$ circuits sorted in the previous step, find $k$ new topologies. Each new topology is found by repeating step 1 but making the addition of one of the $k$ circuits unviable. This process can be easily implemented by temporarily increasing the cost of the analyzed circuit to avoid it being chosen.

This strategy allows the generation of $(k + 1)$ different topologies for the mathematical model and the CHA chosen. The diverse topologies can be increased by means of small modifications made in the iterative process such as: (1) the modification of the sensibility indicator when there are alternative proposals, (2) elimination of irrelevant circuits that are usually added by the CHA algorithm by means of tests that simulate the withdrawal of each circuit added, and (3) stopping the process before the stopping criterion of the CHA is met, choosing an alternative stopping criterion.

Previous proposals allow the finding of a great diversity of quality topologies with topological differences. These topologies can make part of an excellent quality initial population for the GA. For the multi-stage and coordinated planning problem,
algorithms by Garver [1] and by Villasana-Garver [3] are the most efficient because in each case a LP problem that presents an investment alternative is solved with non-integer investment variables for all the planning stages.

D. Specialized Implementation of the GA

In order to improve the performance of a GA it is necessary to incorporate some specialized implementations that take into account the specific characteristics of the problem. Without them, a GA can hardly find sub-optimum solutions to large and complex systems. The main implementations of the GA for the problem of transmission systems expansion planning are presented in a simplified way.

1) Building Blocks and Unconnected Networks: One of the main problems when finding sub-optimum topologies in the planning problem occurs when we have an electric system with a high isolation level, which means that there are many isolated buses in the base topology. In this type of systems, very often, many circuits must be added simultaneously to find a good investment alternative. To illustrate this problem, figure 2 shows a load bus of 700 MW and two alternative paths to supply this demand. Additionally, suppose that in the optimum topology the circuits shown in 2b are added. Accordingly, we must note that: (1) several circuits must be added to different paths to solve the problem of demand of the 700 MW bar, (2) the combination of additions in the 3 paths can be considered as excellent or very bad, (3) random additions can produce inconsistent topologies like those including circuits in path 9-96 and omitting paths 3-9 and 9-71.

![Figure 2: Building blocks in planning problem](image)

To solve this problem, a strategy for new paths was developed whereby all circuits on new paths around a generation or demand zone are considered as a unique building block, which cannot be destroyed by recombination and must be modified properly by mutation. For instance, in Figure 2 the circuits added to the paths 3-9, 9-71 and 9-96, which are generally found in separate locations of the codification vector (see figure 2), are reviewed for a unique offspring, regardless of the recombination point. Consequently, appropriate pointers identify the sets of circuits that form a type of ideal building block that cannot be destroyed by recombination. Those special structures are multiplied or eliminated by the selection operator, are not modified by the recombination operator and are generated or destroyed by the mutation operator. This kind of strategy also eliminates the possibility of generating inappropriate topologies.

The building blocks strategy discussed previously enables the addition and/or withdrawal of a set of circuits simultaneously, resulting in an efficient searching process of alternative paths in the planning process. Without this strategy, the GA converges, generally prematurely, to a local optimum.

2) Variable Parameter $\alpha$: The value of the parameter $\alpha$ should be variable. The value of $\alpha$ determines the influence of the unfeasibility (loss of load) of the objective function. Small values of $\alpha$ increase the possibility of selecting unfeasible topologies whereas large values of $\alpha$ result in the elimination of all the unfeasible topologies. Therefore, $\alpha$ must be assigned the appropriate values. Generally, an efficient strategy should be to choose a relatively small $\alpha$ in the initial stages of the optimization process giving the unfeasible topologies higher survival probability (especially when the initial population is generated as proposed in this article). During the process, the value of $\alpha$ must be increased to give priority to the feasible topologies.

3) Variable Mutation Rate: The mutation rate of a GA can also be variable. In the initial stages of the process, a low mutation rate works adequately since it produces small changes in the topologies of the population. However, in the final stages of the process the homogeneity of the topologies increases and a high mutation rate may produce significant disturbances to the topologies of the system, giving the optimization process an additional dynamism. Therefore, the mutation rate should increase during the optimization process.

4) Mutation Rate Controlled by Simulated Annealing: A strategy that is also used by researchers consists in using a mutation controlled by SA or by an equivalent strategy. The proposal consists basically in carrying out an intelligent mutation. In this way, if a mutation proposal yields an objective function of better quality, then the mutation should be accepted, otherwise the mutation is accepted in a probabilistic way as in the SA algorithm. For the planning problem, the implemented strategy is as follows: (1) if the current topology has no loss off load then simulate the withdrawal of a circuit and if the withdrawal of the circuit does not produce a loss off load then accept the mutation, otherwise accept the mutation in a probabilistic way, and (2) if the current topology has a loss off load then simulate the addition of a circuit, and if the addition of the circuit improves the objective function of the new topology then accept the mutation; otherwise accept the mutation in a probabilistic way.

To implement the mutation rate proposal controlled by SA, the initial temperature must be calculated or estimated and this parameter must be recalculated during the process. An LP prob-
lem should be solved to check whether the new topology has better quality than the current one. Consequently, this topology increases the processing time.

5) Independent codification: In the codification proposed in Figure 1 it is explicit that a circuit added in a planning stage should be used in the following planning periods. That proposal seems to be natural and completely logical. However, in the planning of highly meshed systems the addition of a new circuit does not necessarily improve the operation of a system, contradicting what would be logically expected. This kind of behavior is known as non-coherent behavior and appears frequently in real systems. To allow this behavior an independent codification for each planning period can be used as shown in Figure 3.

![Figure 3: Alternative codification](http://www.dsee.fee.unicamp.br/planning.pdf)

When the alternative codification illustrated in Figure 3 is used, it is possible that circuits added in one planning stage may not be used in all the subsequent planning stages, that is, some circuits added in a planning stage may be withdrawn in subsequent planning stages. In this way, for instance, in the codification shown in Figure 3, two circuits are added to the path 1−2 in $P_{a1}$ and one additional circuit in $P_{a2}$, amounting to 3 circuits operating in $P_{a2}$. If on the other hand, three circuits are added to the path 2−4 in $P_{a1}$, whereas in $P_{a2}$ one of those circuits is withdrawn, as a result only two of the circuits added in $P_{a1}$ operate in $P_{a2}$. The objective function should be properly calculated considering the period during which each circuit is added and withdrawn, with a withdrawal cost of a previously added circuit equal or different to zero.

The proposed codification may perform less efficiently in coherent electric systems but may find better-quality topologies in non-coherent systems.

### IV. Specialized Genetic Algorithm

In this paper, the following specialized GA was developed and used:

1) Choose the population size $n_p$, the number of planning stages $T$ and find the initial population by using relaxed models and CHA. Choose the stop criterion where $k_{max}$ is the maximum number of generations in which the best solution does not improve and $k_{stop}$ is the maximum number of generations of the GA.

2) For each topology of the population calculate its fitness using the LP algorithm. If possible, update the incumbent.

3) If the number of executed generations $k_{g} > k_{max}$, stop the process. Otherwise go to step 4.

4) Implement selection using either proportional selection or tournament selection and limiting the maximum number of offspring of each topology.

5) Implement recombination using either one or two-point recombination.

6) Implement mutation, single or double, controlling it with the SA logic and return to step 2.

### V. Tests and Results

The specialized GA was used to analyze several electric systems. The results of the following two large systems are presented: (1) the Colombian system of medium complexity, and (2) the north-northeast Brazilian system of high complexity. In the results only the added circuits are presented, i.e., the number of $n_{ij}$ circuits added during optimization process. The circuits present in base topology can be found in [http://www.dsee.fee.unicamp.br/planning.pdf](http://www.dsee.fee.unicamp.br/planning.pdf)

#### A. Colombian System

The topology of this system is illustrated in Figure 4 and its data is available from the authors. In figure 4 the lines represent existing circuits in the base topology and the traced lines represents new paths for circuit addition, as well as the addition of parallel circuits to the existing one is permitted. The available data allows a 3 stage planning, namely $P_1$, $P_2$ and $P_3$.

The best topology found, with the actual value of investment projected to the base year 2002 equal to $v = 514.4$ million dollars and loss of load $w = 0$ MW, for the three operation stages, is the following:

- **Stage $P_1$:** 2002-2005: $v_1 = 1.00 \times 316.44 \times 10^6$ US $n_{45-81} = 1$, $n_{55-57} = 1$, $n_{55-62} = 1$, $n_{58-57} = 1$, $n_{68-85} = 1$.
- **Stage $P_2$:** 2005-2009: $v_2 = 0.729 \times 167.37 \times 10^6$ US $n_{19-82} = 1$, $n_{27-29} = 1$, $n_{56-57} = 1$, $n_{62-73} = 1$, $n_{72-73} = 1$.
- **Stage $P_3$:** 2009-2012: $v_3 = 0.478 \times 158.79 \times 10^6$ US $n_{15-18} = 1$, $n_{19-82} = 1$, $n_{29-64} = 1$, $n_{30-65} = 1$, $n_{30-72} = 1$, $n_{43-88} = 2$, $n_{55-84} = 1$, $n_{68-86} = 1$.

The optimization process also finds several good-quality topologies very close to the best topology found. A typical test for the Colombian system solves about 210000 LP problems. The following parameters were used: $\alpha = [1200 - 1400]$, annual discount rate $I = 10\%$, initial population size $n_p \times T = [200-400]$, maximum number of generations $g_{max} = [200-600]$, recombination rate $\rho_r = 0.9$, maximum number of circuits per path $n_{max} = 5$, mutation rate $\rho_m = [0.01 - 0.03]$, maximum number of offspring per topology $t_p = 3$, number of participating topologies in each game $k = 3$, initial temperature for SA $T_0 = [30000 - 100000]$ with temperature reduction rate $\beta = 0.997$ and the process ends if the best solution does not improve after 100 generations. Obviously, the circuits added to $P_1$ appear in the objective function with their nominal costs and those added to $P_2$ and $P_3$ are multiplied by 0.729 and 0.478 respectively.
B. Brazilian North-northeast System

The data for this system are available from the authors. They were not updated in order to permit comparative analysis. In this way, the base topology (dating back to 1990) and the data of the Plan 1 (2002) and Plan 2 (2008), which are simply called $P_1$ and $P_2$ respectively, are available. For the tests, 1998 was used as the base year and periods $P_1$ and $P_2$ as 2002 and 2008 respectively.

The topology of this system is shown in [5]. In this case, tests with two types of codification were conducted (See Figures 1 and 3). The best topology was found using the codification of Figure 1, with the actual value of the investment projected to the base year 1998 equal to $v = 2, 204.28$ million dollars and loss off load $w = 0$ MW, for the two operation periods, with the following topology:

- Stage $P_1$: 1998-2002: $v_1 = 1.00 \times 1,429.21 \times 10^6$ $\$US$

  \begin{align*}
    n_{25-55} &= 1, \quad n_{26-54} = 1, \quad n_{29-30} = 1, \quad n_{30-31} = 1, \\
    n_{35-51} &= 1, \quad n_{36-39} = 1, \quad n_{36-46} = 1, \quad n_{43-55} = 1, \\
    n_{43-58} &= 1, \quad n_{48-50} = 1, \quad n_{49-50} = 2, \quad n_{52-59} = 1, \\
    n_{61-85} &= 1, \quad n_{65-66} = 1, \quad n_{65-87} = 1, \quad n_{73-74} = 1, \\
    n_{75-81} &= 1.
  \end{align*}

The optimization process finds thousands of good-quality topologies close to the best topology found. A typical test for the Brazilian system solves about 330000 LP problems. The following parameters were used: $\alpha = [800 - 100], \beta = 10\%$, $n_p \times T_e = [200-400], g_{max} = [200 - 600], p_r = 0.9, n_{max} = 14, \rho_m = [0.01 - 0.03], \gamma_p = 2, k = 3, T_n = [30000 - 100000]$ with $\beta = 0.997$ and stopping if the best solution does not improve after 100 generations. Obviously, the circuits added in $P_1$ appear in the objective function with their nominal values and 0.656 multiplies those added in $P_2$.

An alternative way of solving the planning problem consists of using a static planning algorithm as presented in [6] and then applying twice the same algorithm for the consecutive stages. Therefore, using the base topology, the planning horizon $P_1$ is solved with static algorithm, then circuits added in that stage are incorporated in the base topology for solving the $P_2$ stage, using the $P_1$ stage results as reference. Applying this methodology, with two consecutive static process the following topology is found:

- Stage $P_1$: 1998-2002: $v_1 = 1.00 \times 1,360.961 \times 10^6$ $\$US$

  \begin{align*}
    n_{20-21} &= 1, \quad n_{20-38} = 1, \quad n_{20-66} = 1, \quad n_{22-58} = 1,
  \end{align*}

  \begin{align*}
    n_{22-23} &= 1, \quad n_{22-58} = 3, \quad n_{23-24} = 1, \quad n_{25-55} = 2, \\
    n_{26-29} &= 1, \quad n_{27-53} = 1, \quad n_{29-30} = 1, \quad n_{34-39} = 1, \\
    n_{34-41} &= 2, \quad n_{36-46} = 1, \quad n_{39-42} = 1, \quad n_{39-86} = 3, \\
    n_{40-45} &= 1, \quad n_{42-44} = 2, \quad n_{42-85} = 1, \quad n_{48-49} = 2, \\
    n_{49-50} &= 1, \quad n_{52-59} = 1, \quad n_{53-86} = 1, \quad n_{54-55} = 1, \\
    n_{54-58} &= 1, \quad n_{67-68} = 1, \quad n_{67-69} = 1, \quad n_{67-71} = 3, \\
    n_{71-72} &= 1, \quad n_{72-73} = 1, \quad n_{73-74} = 1.
  \end{align*}

- Stage $P_2$: 2002-2008: $v_2 = 0.656 \times 1,393.16 \times 10^6$ $\$US$

  \begin{align*}
    n_{01-02} &= 1, \quad n_{04-05} = 3, \quad n_{04-81} = 3, \quad n_{05-56} = 1, \\
    n_{05-58} &= 2, \quad n_{13-14} = 1, \quad n_{13-15} = 2, \quad n_{15-16} = 1, \\
    n_{16-18} &= 1, \quad n_{16-18} = 2, \quad n_{16-18} = 2, \quad n_{16-18} = 2, \\
    n_{16-44} &= 3, \quad n_{18-50} = 1, \quad n_{18-74} = 3, \\
    n_{20-21} &= 1, \quad n_{20-38} = 1, \quad n_{20-66} = 1, \quad n_{22-58} = 1,
  \end{align*}

with the actual value of the investment projected to the base year 1998 equal to $v = 2,274.88$ million dollars. The multi-stage planning strategy finds a topology with investment 70.6 million dollars less than the topology found with two consecutive static planning approaches. The result was expected since the multi-stage planning executes an integral investment strategy, making a simultaneous analysis of all stages instead of static planning, which consider only one stage.
C. Comments on the Results

The specialized GA for the problem of multi-stage and coordinated planning of expansion transmission systems yields excellent results in the two systems analyzed. However, since there are no articles in the specialized literature on multi-stage and coordinated planning and no tests have been reported with the systems analyzed using other optimization methods, there is no conclusive proof of the optimality of the results found. This is particularly true for the Brazilian north-northeast system that is very complex. Therefore, the topologies found can be only considered best-quality topologies or sub-optimum topologies. Nevertheless, there are articles in the literature where these systems are studied using transmission expansion static planning techniques, [6], [8], [9], [16], [17] and compared to them, coordinated multi-stage planning finds better solutions, because it spreads investment over the whole horizon by proposing an orderly investment in the time.

VI. CONCLUSIONS

A specialized GA algorithm for solving the problem of multi-stage and coordinated planning of transmission systems expansion was developed. This GA presents several specialization levels such as the codification proposal, the efficient generation of the initial population, the structure of special building blocks, variable control parameters, specialized codification, etc. The results found by analyzing large and complex systems, show high-quality topologies and an excellent performance of the specialized GA.

The multi-stage coordinated planning of expansion transmission systems has as its main characteristic that of adapting to the continuous growth of the demand and generation, in contrast with static planning, that considers only the initial and final year’s demand and generation. Multi-stage planning invests in the proper time and quantity. Besides, multi-stage planning takes advantage of economies of scale, because it favors the addition of large-capacity expensive elements necessary in the long term, which are ruled out by static planning, which favors the immediate reinforcements in the transmission system.

VII. ACKNOWLEDGMENTS

This research is partially supported by CNPq, FAPESP in Brasil and by Universidad Tecnologica de Pereira in Colombia. The authors wish to thank these institutions for their support to the Electric Planning Work Group and would also like to express their gratitude to Professor Alcir Monticelli (in memoriam) for his priceless assistance and academic encouragement. Thanks are also due to Herman Serrano and Julio César Jaramillo from UTP for their useful suggestions.

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